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Chaos synchronization in a 6-D hyperchaotic system with self-excited attractor

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ABSTRACT

This paper presented stability application for chaos synchronization using a 6-D hyperchaotic system of different controllers and two tools: Lyapunov stability theory and Linearization methods. Synchronization methods based on nonlinear control strategy is used. The selecting controller's methods have been modified by applying complete synchronization. The Linearization methods can achieve convergence according to the of complete synchronization. Numerical simulations are carried out by using MATLAB to validate the effectiveness of the analytical technique.

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1. INTRODUCTION

In recent years, the dynamical system has attracted significant attention due to its widespread applications in engineering and different scientific research as lasers, nonlinear circuits biological [1, 2], engineering [3, 4] and secure communications [5, 6]. Lorenz system is the first physical and mathematical model of a chaotic system contains real variables only which discovered in 1963 and open the way to find another chaotic system such as Chen system, Lu system, Liu system and Pan system [7-9]. Each system has a 3-D of differential equations and just one positive Lyapunov exponent [10]. One important application in the field of engineering is secure communication i.e., the messages which are made by such simple chaotic systems are not always safe [6, 11, 12]. It is suggested that this problem can be overcome by using higher-dimensional hyperchaotic systems, which have increased randomness and higher unpredictability.

In 1979, Rössler discovers the first 4-D hyperchaotic system including real variables with two positive Lyapunov exponents and followed to discover another 4-D, as well as 5-D hyperchaotic with three positive Lyapunov exponents [10, 13-15] and some other systems, have been revealed. The dynamical systems with higher dimensions are effective and interesting compared with the low dimensions [16-18]. In 2015, Yang et al., proposes a 6-D hyperchaotic system including real variables and has four positive Lyapunov exponents [19].

These days, the synchronization of the mentioned systems witnessed large attention by researchers because of its important applications in the is secure communication [20-22]. Many of the papers that relate to this topic are increasing, and numerous research devoted to investigating CS of high-dimensional hyperchaotic systems based on traditional Lyapunov stability theory [23-25]. Lyapunov stability theory is extensively utilized in the phenomena of synchronization because the Lyapunov function can deliver accurately and speed data of the system convergence. However, Lyapunov function in some time is incapable of meeting the convergence requirements of error dynamics system owing to suffers from its drawbacks of modified the function itself. To achieve synchronization of good performance, the Linearization tool is preferred. So the Linearization and nonlinear control strategy integration can achieve higher performance.

The contributions of this research can be summarized in the following points.

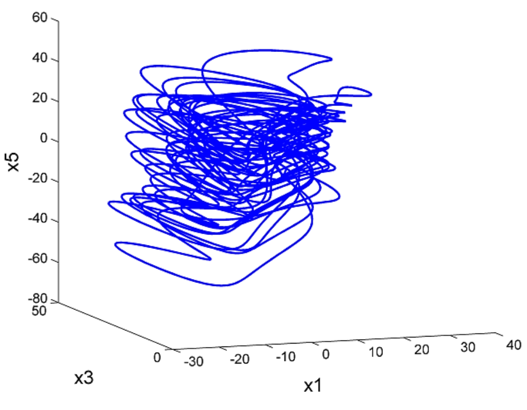
a. Chaos synchronization between identical 6-D hyperchaotic systems is studied and used to find the error dynamics between them and its secure communication is then presented theoretically.
b. Designs of three different controllers of complete synchronization are done by a nonlinear control strategy based on the Lyapunov stability theory, Linearization method.
c. Compare between the Lyapunov and Linearization method.

2. SYSTEM DESCRIPTION

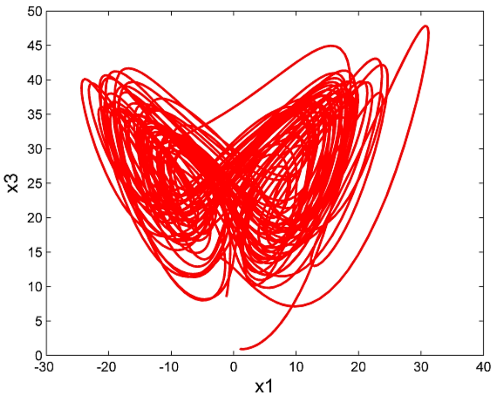
The Lorenz system was the first 3-D chaotic system to be modeled and one of the most widely studied. The original system was modified into a 4-D and 5-D hyperchaotic systems by introducing a linear feedback controller. In 2015, Yang constructed a 6-D hyperchaotic system which contains four positive Lyapunov Exponents $LE_1 = 1.0034$, $LE_2 = 0.57515$, $LE_3 = 0.32785$, $LE_4 = 0.020937$, and two negative Lyapunov Exponents $LE_5 = -0.12087$, $LE_6 = -12.4713$. The 6-D system which is described by the following mathematical form [19]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3 + x_5 \\ \dot{x}_3 = -bx_3 + x_1x_2 \\ \dot{x}_4 = dx_4 - x_1x_3 \\ \dot{x}_5 = -kx_2 \\ \dot{x}_6 = hx_6 + rx_2 \end{cases} \tag{1}$$

where $x_1, x_2, x_3, x_4, x_5, x_6$ are real state variables and a, b, c, d, k, h, r are all positive real parameters which equals $(10, 8/3, 28, 2, 8.4, 1, 1)$ respectively. This system is rich in dynamic properties. Figure 1 (a) shows the 3-D attractor of the system (1), while Figure 1 (b) shows the 2-D attractor of the same system.



(a)



(b)

Figure 1. The attractor of the system (1), (a) In the 3-D(x_1, x_3, x_6) space, (b) In the 2-D (x_1, x_3) plane

3. CHAOS SYNCHRONIZATION BETWEEN TWO IDENTICAL LORENZ SYSTEM

In this section, two systems are needed, the first system is called the drive system which represents the picture or message information will be sent while the second system is called response system represents the noise that followed this information to ensure that they are not penetrated. Assume that the system (1) is the drive system and can be written as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix}=\underbrace{\begin{bmatrix}-a & a & 0 & 1 & 0 & 0\\ c & -1 & 0 & 0 & 1 & 0\\ 0 & 0 & -b & 0 & 0 & 0\\ 0 & 0 & 0 & d & 0 & 0\\ 0 & -k & 0 & 0 & 0 & 0\\ 0 & r & 0 & 0 & 0 & h\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix}+\underbrace{\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}-x_1x_3\\ x_1x_2\\ -x_1x_3\end{bmatrix}}_{C} \quad (2)$$

A and the product $B.C$ represents parameters matrix and nonlinear part of the system (1), respectively. While the response system is as follows:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\\ \dot{y}_4\\ \dot{y}_5\\ \dot{y}_6\end{bmatrix}=A_1\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\\ y_5\\ y_6\end{bmatrix}+\left(B_1\underbrace{\begin{bmatrix}-y_1y_3\\ y_1y_2\\ -y_1y_3\end{bmatrix}}_{C_1}+\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\end{bmatrix}\right) \quad (3)$$

and let $U=[u_1,u_2,u_3,u_4,u_5,u_6]^T$ is the nonlinear controller to be designed. The synchronization error dynamics between the 6-D hyperchaotic system (2) and system (3) is defined as $e_i=y_i-x_i$, $i=1,2,\dots,6$ and satisfied that, $\lim_{t\to\infty} e_i=0$. The error dynamics is calculated as the following:

$$\begin{cases}\dot{e}_1=a(e_2-e_1)+e_4+u_1\\ \dot{e}_2=\mathrm{c}e_1-e_2-e_1e_3-x_3e_1-x_1e_3+e_5+u_2\\ \dot{e}_3=-\mathrm{b}e_3+e_1e_2+x_2e_1+x_1e_2+u_3\\ \dot{e}_4=\mathrm{d}e_4-e_1e_3-x_3e_1-x_1e_3+u_4\\ \dot{e}_5=-ke_2+u_5\\ \dot{e}_6=he_6+re_2+u_6\end{cases} \quad (4)$$

If the matrices A_1 and B_1 as
$A_1=A$ and $B_1=B$, then refer for identical synchronization.
$A_1\neq A$ or $B_1\neq B$, then refer for non-identical synchronization.
Based on Linearization method, The system (4) is unstable and the characteristic equation and eigenvalues are respectively as

$$\lambda^6+\tfrac{32}{3}\lambda^5-\tfrac{4069}{15}\lambda^4+\tfrac{1658}{15}\lambda^3+\tfrac{24004}{15}\lambda^2-\tfrac{9496}{5}\lambda-448=0$$

$$\begin{cases}\lambda_1=2\\ \lambda_2=1\\ \lambda_3=-8/3\\ \lambda_4=11.3659-8.10^{-9}i\\ \lambda_5=-22.6916-3.92820323010^{-9}i\\ \lambda_6=0.3257+9.92820323010^{-9}i\end{cases}$$

Now, different controllers are designed based on Lyapunov and Linearization methods and we compare them.
Theorem 1. If the control U of system (4) is design as the following:

$$\begin{cases}u_1=e_4(x_3-1)-e_2(\mathrm{a}+c-x_3)\\ u_2=-re_6\\ u_3=-x_2e_1\\ u_4=e_3(e_1+x_1)-3de_4\\ u_5=-e_2(1-k)-e_5\\ u_6=-2he_6\end{cases} \quad (5)$$

Then the system (3) can be followed by the system (2) by two methods.
Proof. Substitute above control in the error dynamics system (4) we have (6).

$$\begin{cases} \dot{e}_1 = -ae_1 + x_3e_4 - ce_2 + x_3e_2 \\ \dot{e}_2 = ce_1 - e_2 - e_1e_3 - x_3e_1 - x_1e_3 + e_5 - re_6 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_1e_2 \\ \dot{e}_4 = -2de_4 - x_3e_1 \\ \dot{e}_5 = -e_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \tag{6}$$

In the first method (Linearization method), the characteristic equation and eigenvalues as

$$\lambda^6 + \frac{32}{3}\lambda^5 + \frac{2488}{3}\lambda^4 + \frac{20696}{3}\lambda^3 + \frac{59225}{3}\lambda^2 + \frac{66172}{3}\lambda + \frac{25184}{3} = 0$$

$$\begin{cases} \lambda_1 = -4 \\ \lambda_2 = -1 \\ \lambda_3 = -1 \\ \lambda_4 = -8/3 \\ \lambda_5 = -1 + \sqrt{786}i \\ \lambda_6 = -1 - \sqrt{786}i \end{cases}$$

All real parts of eigenvalues are negative, the linearization method is realized the chaos synchronization between system (2) and system (3). If the Lyapunov function is constructed as (7).

$$V(e_i) = \frac{1}{2}\sum_{i=1}^{6} e_i^2 = e_i^T Pe_i \quad , \quad P = daig(0.5, 0.5, 0.5, 0.5, 0.5, 0.5) \tag{7}$$

The derivative of the above function $V(e_i)$ is

$$\dot{V}(e_i) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_5\dot{e}_5 + e_6\dot{e}_6$$

$$\dot{V}(e_i) = e_1(-ae_1 + x_3e_4 - ce_2 + x_3e_2) + e_2(ce_1 - e_2 - e_1e_3 - x_3e_1 - x_1e_3 + e_5 - re_6) + e_3(-be_3 + e_1e_2 + x_1e_2) + e_4(-2de_4 - x_3e_1) + e_5(-e_2 - e_5) + e_6(re_2 - he_6)$$

$$\dot{V}(e_i) = -ae_1^2 - e_2^2 - be_3^2 - 2de_4^2 - e_5^2 - he_6^2 = -e_i^T\, Q\, e_i \tag{8}$$

where $Q = diag(a, 1, b, 2d, 1, h)$, so $Q > 0$. Consequently, $\dot{V}(e_i)$ is negative definite on R^6. The nonlinear controller is suitable and the complete synchronization is achieved. Now, we will take the initial values as $(1,0,2,4,1,-1)$ and $(-8,-7,-15,12,20,1)$ to illustrate the complete synchronization that happened between (2) and (3) numerically. Figure 2 shows verify these results numerically.

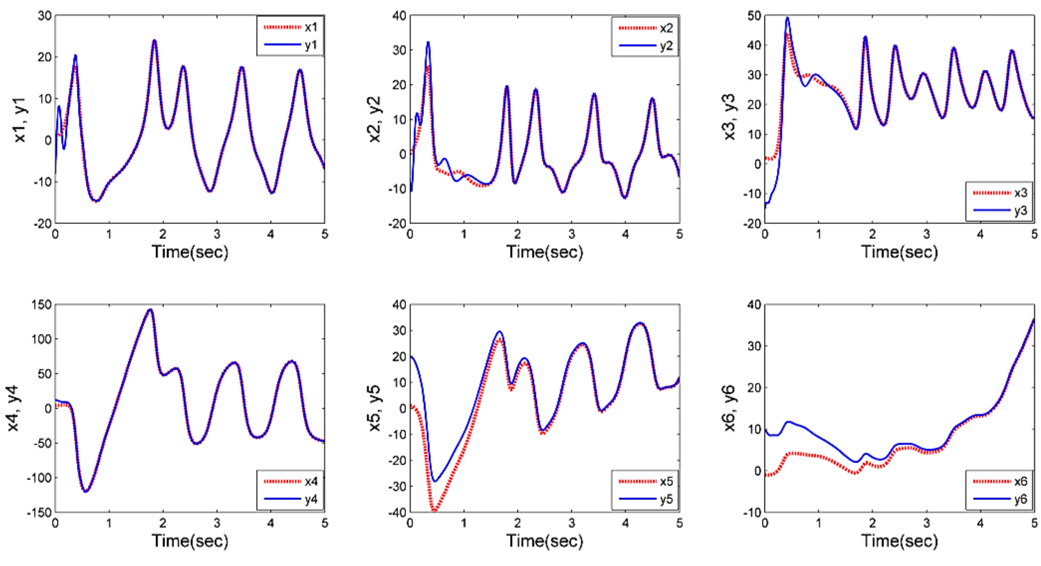


Figure 2. Complete synchronization between systems (2) and (3) with control (5)

Theorem 2. If the nonlinear control U of error dynamical system (4) is designed (9).

$$\begin{cases} u_1 = -ce_2 - x_2e_3 + x_3(e_4 + e_2) \\ u_2 = -ae_1 - re_6 \\ u_3 = x_1e_4 \\ u_4 = e_1(e_3 - d) - 2de_4 \\ u_5 = -e_5 \\ u_6 = -2he_6 \end{cases} \tag{9}$$

Then the system (3) can be followed by the system (2) by two methods.
Proof. From the above control (9) with the error system (4), we get (10).

$$\begin{cases} \dot{e}_1 = ae_2 - ae_1 + e_4 - ce_2 - x_2e_3 + x_3e_4 + x_3e_2 \\ \dot{e}_2 = ce_1 - e_2 - e_1e_3 - x_3e_1 - x_1e_3 + e_5 - ae_1 - re_6 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_2e_1 + x_1e_2 + x_1e_4 \\ \dot{e}_4 = -de_4 - x_3e_1 - x_1e_3 - de_1 \\ \dot{e}_5 = -ke_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \tag{10}$$

Based on the first method (Linearization method), the characteristic equation and eigenvalues as:

$$\lambda^6 + \frac{53}{3}\lambda^5 + \frac{2172}{5}\lambda^4 + \frac{38594}{15}\lambda^3 + \frac{91112}{15}\lambda^2 + \frac{93856}{15}\lambda + \frac{35072}{15} = 0$$

$$\begin{cases} \lambda_1 = -1 \\ \lambda_2 = -8/3 \\ \lambda_3 = -1.3438 \\ \lambda_4 = -1.9026 \\ \lambda_5 = -5.3768 + 17.7207\,i \\ \lambda_6 = -5.3768 - 17.7207\,i \end{cases}$$

all real parts of eigenvalues are negative. The linearization method is succeeded to achieve complete synchronization. In Lyapunov approach, the Lyapunov function is taken as the same form in theorem1, the derivative Lyapunov function with control (9) becomes

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2 + e_1e_4(1-d) + e_2e_5(1-k) = -e^TQ_1\,e \tag{11}$$

where

$$Q_1 = \begin{bmatrix} a & 0 & 0 & -(1-d)/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -(1-k)/2 & 0 \\ 0 & 0 & b & 0 & 0 & 0 \\ -(1-d)/2 & 0 & 0 & d & 0 & 0 \\ 0 & -(1-k)/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & h \end{bmatrix}$$

Note that Q_1 is not a diagonal matrix. If all the following five inequalities are satisfied, then the Q_1 is positive definite:

$$\begin{cases} 1.\ a > 0 \\ 2.\ b > 0 \\ 3.\ h > 0 \\ 4.\ \left(ad - \frac{(1-d)^2}{4}\right) > 0 \\ 5.\ \left(ad\left(1 - \frac{(1-k)^2}{4}\right) - \frac{(1-d)^2}{4}\left(1 - \frac{(1-k)^2}{4}\right)\right) > 0 \end{cases} \tag{12}$$

Fifth inequality is not correct with given parameters. Therefore, this control is failed. If update the matrix P with the same control as:

$$P_1 = diag(1/2, 1/2, 1/2, 1/4, 5/84, 1) \tag{13}$$

Then, the derivative of Lyapunov function as:

$$\dot{V}(e_i) = -10e_1^2 - e_2^2 - \frac{8}{3}e_3^2 - e_4^2 - \frac{5}{42}e_5^2 - e_6^2 = -e^T Q_2 e \tag{14}$$

where $Q_2 = diag(10,1,8/3,1,5/42,1)$ is a positive definite. Figure 3 shows verify these results numerically.

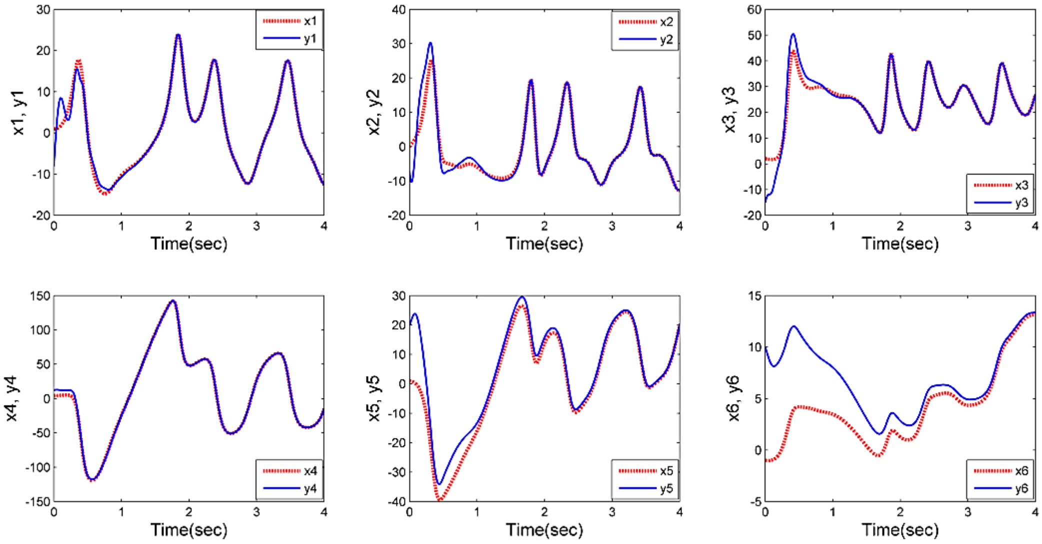


Figure 3. Complete synchronization between systems (2) and (3) with control (9)

Theorem 3. If the nonlinear control U of error dynamical system (4) is designed as:

$$\begin{cases} u_1 = -ce_2 - a(e_5 + e_2) \\ u_2 = -re_6 + x_3 e_1 \\ u_3 = e_4(x_1 + e_1) - x_2 e_1 \\ u_4 = -e_1 - 2de_4 + x_3 e_1 \\ u_5 = -e_2 - e_5 + k(2e_1 + e_2) \\ u_6 = -2he_6 \end{cases} \tag{15}$$

then the system (3) can be followed by the system (2) by linearization method only.
Proof. Rewrite system (4) with control (15) as follows (16).

$$\begin{cases} \dot{e}_1 = -ae_1 + e_4 - ce_2 - ae_5 \\ \dot{e}_2 = ce_1 - e_2 - e_1 e_3 - x_1 e_3 + e_5 - re_6 \\ \dot{e}_3 = -be_3 + e_1 e_2 + x_1 e_2 + x_1 e_4 + e_1 e_4 \\ \dot{e}_4 = -de_4 - e_1 e_3 - x_1 e_3 - e_1 \\ \dot{e}_5 = -e_2 - e_5 + 2ke_1 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \tag{16}$$

Based on the Lyapunov stability theory, we obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2 + e_1 e_5(2k - a) = -e^T Q_3 e \tag{17}$$

where

$$Q_3 = \begin{bmatrix} a & 0 & 0 & 0 & -(a-2k)/2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ (a-2k)/2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & h \end{bmatrix} \tag{18}$$

So Q_3 is not a diagonal matrix. The necessary conditions to make Q_3 is positive definite, the following inequalities must hold.

$$\begin{cases} 1.\ a > 0 \\ 2.\ b > 0 \\ 3.\ d > 0 \\ 4.\ h > 0 \\ 5.\ a > \frac{(a-2k)^2}{4} \end{cases} \tag{19}$$

Note all inequalities are realized except the fifth inequality. So, the matrix Q_3 is a negative definition, and failed to achieve complete synchronization. Therefore modified the matrix P as follows:

$$\begin{cases} P_{3,1} = diag(21/25, 1/2, 1/2, 1/2, 1/2, 1/2) \\ P_{3,2} = diag(1/2, 1/2, 1/2, 1/2, 25/84, 1/2) \\ P_{3,3} = diag(1/20, 1/2, 1/2, 1/2, 5/168, 1/2) \end{cases}$$

all the above matrices are not diagonal Q_3, therefore Lyapunov method failed. Based on Linearization method, the characteristic equation and eigenvalues as

$$\lambda^6 + \frac{53}{3}\lambda^5 + 1054\lambda^4 + \frac{34142}{5}\lambda^3 + \frac{83193}{5}\lambda^2 + \frac{53173}{3}\lambda + \frac{35784}{5} = 0$$

$$\begin{cases} \lambda_1 = -8/3 \\ \lambda_2 = -1.9967 \\ \lambda_3 = -1.1097 - 0.4060\,i \\ \lambda_4 = -1.1097 + 0.4060\,i \\ \lambda_5 = -5.3920 - 30.5554\,i \\ \lambda_6 = -5.3920 + 30.5554\,i \end{cases}$$

Note that all eigenvalues with negative real parts, and thus the Linearization method has succeeded in achieving complete synchronization between systems (2) and (3) without any update compared to the Lyapunov method and thus the proof has been completed. These results are justified numerically in Figure 4.

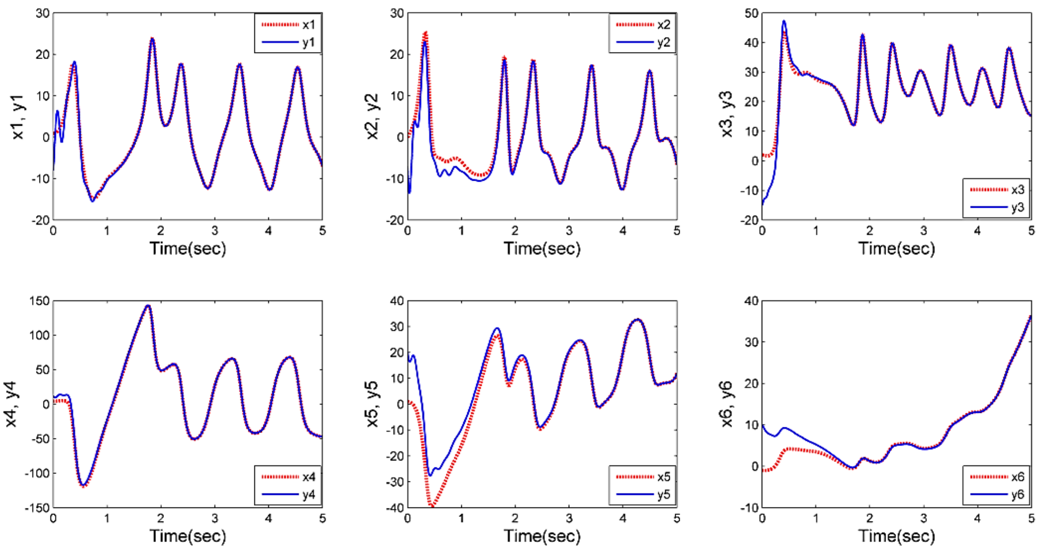


Figure 4. Complete synchronization between systems (2) and (3) with control (15)

4. CONCLUSION

In this paper, complete synchronization of a 6-D hyperchaotic system with a self-excited attractor is proposed. based on nonlinear control strategy and two analytical methods; first is Lyapunov's, and the second is the Linearization method. Through these two approaches we have found the difference between them and what is the appropriate method in each approach for achieving complete synchronization and thus we showed the best way observed that the Linearization method does not need to a auxiliary function or modifying this function as a method Lyapunov. Thus the linearization method is better than the Lyapunov method in achieving the desired one. Numerical results have been found to be the same results as we proposed.

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